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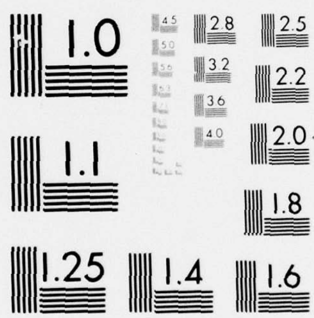

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29th-ORDER HARMONICS IN THE GEOPOTENTIAL FROM THE ORBIT OF ARIEL I AT 29:2 RESONANCE

by

Doreen M. C. Walker

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SUMMARY

In analysing the orbit of Ariel 1 to determine upper-atmosphere winds, it was observed that the orbital inclination underwent a noticeable perturbation in November 1969 at the 29:2 resonance with the Earth's gravitational field, when the satellite track over the Earth repeats every 2 days after 29 revolutions. The variations in the inclination and eccentricity of the orbit between July 1969 and February 1970 have now been analysed, using 35 US Navy orbits, and fitted with theoretical curves to obtain lumped values of 29th-order harmonic coefficients in the geopotential.

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CONTENTS

1 INTRODUCTION

Ariel 1, 1962-15A, the world's first international satellite, was launched on 26 April 1962 and decayed in the Earth's atmosphere on 24 May 1976 after a lifetime of 5142 days. The satellite was cylindrical in shape with four paddles, had a length of 0.53m, a diameter of 0.58m and a mass of 60kg[1]. The initial perigee and apogee heights were 389km and 1214km respectively, and the inclination was 53.9°.

The orbit of Ariel 1 had an initial period of 100.9 minutes. Between 1962 and 1973 the period decreased by more than 5 minutes, under the action of air drag, and the variation in inclination over these 11 years was analysed to determine upper-atmosphere zonal winds[2]. During this analysis it was noticed that in November 1969 the decay under the action of air drag carried the orbit through the condition of 29:2 resonance, when the tracks over the Earth repeat every two days after 29 revolutions. At the time of this resonance a group of 35 US Navy orbits revealed a substantial perturbation in inclination[2].

The aim of this Report is to evaluate, for the first time, harmonic coefficients of order 29 in the geopotential, from the changes that occur in the inclination and eccentricity near the time of the 29:2 resonance.

2 THE THEORETICAL EQUATIONS NEAR 29:2 RESONANCE

The longitude-dependent geopotential at an exterior point (r,θ,λ) may be written in normalized form[3] as

$$\frac{\mu}{r}\sum_{\ell=2}^{\infty}\sum_{m=1}^{\ell}\left(\frac{R}{r}\right)^{\ell} P_{\ell}^{m}(\cos\theta)\left\{\bar{C}_{\ell m}\cos m\lambda + \bar{S}_{\ell m}\sin m\lambda\right\}N_{\ell m} \quad , \tag{1}$$

where r is the distance from the Earth's centre, θ is co-latitude, λ is longitude (positive to the east), μ is the gravitational constant for the Earth ($398601\,km^3/s^2$), R is the Earth's equatorial radius (6378.1km), $P_{\ell}^{m}(\cos\theta)$ is the associated Legendre function of order m and degree ℓ, and $\bar{C}_{\ell m}$ and $\bar{S}_{\ell m}$ are the normalized tesseral harmonic coefficients, of which those of order $m = 29$ particularly concern us here. The normalizing factor $N_{\ell m}$ is given by[3]

$$N_{\ell m}^{2} = \frac{2(2\ell + 1)(\ell - m)!}{(\ell + m)!} \quad . \tag{2}$$

The rate of change of inclination i caused by a relevant pair of coefficients, $\bar{C}_{\ell m}$ and $\bar{S}_{\ell m}$, near $\beta:\alpha$ resonance may be written[4]

$$\frac{di}{dt} = \frac{n}{\sin i}\left(\frac{R}{a}\right)^{\ell}\bar{F}_{\ell mp}G_{\ell pq}(k\cos i - m)\mathcal{R}\left[j^{\ell-m+1}(\bar{C}_{\ell m} - j\bar{S}_{\ell m})\exp\{j(\gamma\Phi - q\omega)\}\right] , \qquad ......(3)$$

where $\bar{F}_{\ell mp}$ is Allan's normalized inclination function[5], $G_{\ell pq}$ is a function of eccentricity e for which explicit forms are given by Gooding[4], $\mathcal{R}$ denotes 'real part of' and $j = \sqrt{-1}$. The resonance angle Φ is defined by the equation

$$\Phi = \alpha(\omega + M) + \beta(\Omega - \nu) , \qquad (4)$$

where ω is the argument of perigee, M the mean anomaly, Ω the right ascension of the node and ν is the sidereal angle. The indices γ, q, k and p in equation (3) are integers, with γ taking the values 1, 2, 3 ..... and q the values 0, ± 1, ± 2, ... ; the equations linking ℓ, m, k and p are[4]: $m = \gamma\beta$; $k = \gamma\alpha - q$; $2p = \ell - k$.

Here $\beta = 29$ and $\alpha = 2$, and the m-suffix of a relevant $(\bar{C}_{\ell m}, \bar{S}_{\ell m})$ pair is given uniquely by the choice of γ . The values of ℓ to be taken must be such that $\ell \geqslant m$ and $(\ell - k)$ is even. The successive coefficients which arise (for given γ and q) may usefully be gathered together in a lumped form and written as[4]

$$\bar{C}_m^{q,k} = \sum_{\ell} Q_\ell \bar{C}_{\ell m} , \quad \bar{S}_m^{q,k} = \sum_{\ell} Q_\ell \bar{S}_{\ell m} , \qquad (5)$$

where ℓ increases in steps of 2 from its minimum permissible value ℓ_0 , and the Q_ℓ are constant coefficients with $Q_{\ell o} = 1$.

For the 29:2 resonance with $\gamma = 1$, we have $m = 29$ and $k = 2 - q$; thus the affixes [q,k] in equation (5) are [0,2], [1,1] and [-1,3] when $q = 0$, 1 and -1 respectively. Writing only the three terms with $(\gamma,q) = (1,0)$, (1,1) and (1,-1) explicitly, the theoretical variation of inclination given by equation (3) may be written for 29:2 resonance[4,5,6] as

$$\begin{aligned}\frac{di}{dt} = \frac{n}{\sin i}\left(\frac{R}{a}\right)^{29}\Bigg[&\frac{R}{a}(29 - 2\cos i)\bar{F}_{30,29,14}\left\{\bar{S}_{29}^{0,2}\sin\Phi + \bar{C}_{29}^{0,2}\cos\Phi\right\}\\ &+ 16e(29 - \cos i)\bar{F}_{29,29,14}\left\{\bar{C}_{29}^{1,1}\sin(\Phi - \omega) - \bar{S}_{29}^{1,1}\cos(\Phi - \omega)\right\}\\ &+ 12e(29 - 3\cos i)\bar{F}_{29,29,13}\left\{\bar{C}_{29}^{-1,3}\sin(\Phi + \omega) - \bar{S}_{29}^{-1,3}\cos(\Phi - \omega)\right\}\\ &+ \text{terms in } e^{|q|}\,{\cos \atop \sin}(\gamma\Phi - q\omega)\Bigg] \quad . \qquad (6)\end{aligned}$$

Only three terms are given explicitly because it is believed the others are small: terms with $q = \pm 2$ have e^2 as a multiplying factor (where $e \simeq 0.04$ for Ariel 1 at the time of 29:2 resonance), while the terms with $\gamma = 2$ are associated with harmonics of order 58, which should be much smaller than those of order 29.

The three pairs of lumped coefficients $\bar{C}_m^{q,k}$ and $\bar{S}_m^{q,k}$ appearing in equation (6) may be written in terms of the individual geopotential coefficients as[6]

$$\bar{C}_{29}^{0,2} = \bar{C}_{30,29} - \frac{\bar{F}_{32,29,15}}{\bar{F}_{30,29,14}}\left(\frac{R}{a}\right)^2\bar{C}_{32,29} + \frac{\bar{F}_{34,29,16}}{\bar{F}_{30,29,14}}\left(\frac{R}{a}\right)^4\bar{C}_{34,29} - \ldots\ldots, \qquad (7)$$

$$\bar{C}_{29}^{1,1} = \bar{C}_{29,29} - \frac{17\bar{F}_{31,29,15}}{16\bar{F}_{29,29,14}}\left(\frac{R}{a}\right)^2\bar{C}_{31,29} + \frac{18\bar{F}_{33,29,16}}{16\bar{F}_{29,29,14}}\left(\frac{R}{a}\right)^4\bar{C}_{33,29} - \ldots\ldots, \qquad (8)$$

and

$$\bar{C}_{29}^{-1,3} = \bar{C}_{29,29} - \frac{13\bar{F}_{31,29,14}}{12\bar{F}_{29,29,13}}\left(\frac{R}{a}\right)^2\bar{C}_{31,29} + \frac{14\bar{F}_{33,29,15}}{12\bar{F}_{29,29,13}}\left(\frac{R}{a}\right)^4\bar{C}_{33,29} - \ldots\ldots, \qquad (9)$$

and similarly for S, on replacing C by S throughout.

In equations (6) to (9) the numerical values of the three most important $\bar{F}$ functions are:

$\bar{F}_{30,29,14} = 0.247414 \sin^{27} i\,(15\cos i - 1)(1 + \cos i)^2 = 15.256 \times 10^{-3}$ for Ariel 1

$\bar{F}_{29,29,14} = 0.506850 \sin^{28} i\,(1 + \cos i) = 2.0224 \times 10^{-3}$ for Ariel 1

$\bar{F}_{29,29,13} = 0.443494 \sin^{26} i\,(1 + \cos i)^3 = 6.8605 \times 10^{-3}$ for Ariel 1.

The contribution of the (ℓ,m) harmonic to de/dt for $\beta:\alpha$ resonance can be written[4]

$$\frac{de}{dt} = n\left(\frac{R}{a}\right)^{\ell}\bar{F}_{\ell mp}G_{\ell pq}\left\{\frac{q - \frac{1}{2}(k + q)e^2}{e}\right\}\mathcal{R}\left[j^{\ell-m+1}(\bar{C}_{\ell m} - j\bar{S}_{\ell m})\exp\{j(\gamma\Phi - q\omega)\}\right] . \quad (10)$$

For the 29:2 resonance, the theoretical variation of eccentricity given by equation (10) may be written in terms of the same $(\bar{C}_m^{q,k}, \bar{S}_m^{q,k})$ as[4,5]

$$\begin{aligned}\frac{de}{dt} = n\left(\frac{R}{a}\right)^{29}\Bigg[& -\frac{R}{a}\bar{F}_{30,29,14}e(\bar{S}_{29}^{0,2}\sin\Phi + \bar{C}_{29}^{0,2}\cos\Phi) \\ & - 16\bar{F}_{29,29,14}\left\{\bar{C}_{29}^{1,1}\sin(\Phi - \omega) - \bar{S}_{29}^{1,1}\cos(\Phi - \omega)\right\} \\ & + 12\bar{F}_{29,29,13}\left\{\bar{C}_{29}^{-1,3}\sin(\Phi + \omega) - \bar{S}_{29}^{-1,3}\cos(\Phi + \omega)\right\} \\ & + \text{terms in}\left[e^{|q|-1}\left\{q - \tfrac{1}{2}(k + q)e^2\right\}\begin{matrix}\cos \\ \sin\end{matrix}(\gamma\Phi - q\omega)\right]\Bigg] . \quad (11)\end{aligned}$$

Three terms are given explicitly in equation (11), those with $(\gamma,q) = (1,0)$, $(1,1)$ and $(1,-1)$. The main terms are expected to be those with $(\gamma,q) = (1,1)$ and $(1,-1)$ but the $(\gamma,q) = (1,0)$ term is included so that the analysis for e conforms with that for i.

3 ANALYSIS OF THE 29:2 RESONANCE

3.1 Procedure

Near the time of the 29:2 resonance in November 1969, 35 sets of US Navy elements were available for analysis. The THROE computer program developed by Gooding[7,4] provides a least-squares fitting of equation (6), in integrated form, to the observed variation in inclination, and similarly a fitting of equation (11) to the observed variation in eccentricity, to obtain values of the lumped 29th-order harmonic coefficients in the geopotential. The inclination and eccentricity can be fitted simultaneously using the SIMRES computer program[4].

3.2 Analysis of inclination

The 35 values of inclination, i, were cleared of zonal-harmonic and lunisolar perturbations by numerical integration at 1-day intervals using the PROD computer program[8], and then fitted with equation (6) using THROE. A standard deviation of 0.001° was assigned to the 35 values of i, the density scale height H was taken as 58km, appropriate to a height of 420km ($\frac{3}{4}H$ above perigee), and the atmospheric rotation rate, Λ, was taken[2] as 1.1rev/day. THROE can be run for specified pairs of values of (γ,q) and was first run with $(\gamma,q) = (1,0)$, $(1,1)$ and $(1,-1)$, the terms given explicitly in equation (6). For this run the measure of fit, ε, had the value 1.356. Several runs were computed with different (γ,q) terms added and also with Λ changed to 1.2, but all these yielded higher values of ε. So the first choice of values of (γ,q) and Λ proved to be best. In the initial run the first three orbits and the last had values of the resonance angle Φ more than 1600° greater than its value at resonance (which was 76°). So these four outlying values, and one badly-fitting intermediate value (at MJD 40484), were omitted: the resulting fit, with 30 values, was better, with $\varepsilon = 1.135$, and this run was taken as final.

The values of Φ and $\dot{\Phi}$ are shown in Fig.1. The 'main resonance' $\dot{\Phi} = 0$, and the 'subsidiary resonances' corresponding to the extra terms $(\gamma,q) = (1,1)$ and $(1,-1)$, namely $\dot{\Phi} = \dot{\omega}$ and $\dot{\Phi} = -\dot{\omega}$, all occur within 14 days, between 3 and 16 November 1969.

The 30 values of i, cleared of zonal-harmonic and lunisolar perturbations, are plotted in Fig.2 and the theoretical curve given by the final THROE run is shown with a broken line. The values of the C and S coefficients obtained from the final THROE run are:

$$\left.\begin{array}{llll} 10^6\bar{S}_{29}^{0,2} = 1.1 \pm 0.5 & & 10^6\bar{C}_{29}^{0,2} = -0.9 \pm 0.4 \\ 10^6\bar{C}_{29}^{1,1} = -5.6 \pm 3.3 & & 10^6\bar{S}_{29}^{1,1} = -6.5 \pm 4.5 \\ 10^6\bar{C}_{29}^{-1,3} = 4.8 \pm 1.8 & & 10^6\bar{S}_{29}^{-1,3} = 0.3 \pm 1.5 \ . \end{array}\right\} \qquad (12)$$

3.3 Analysis of eccentricity

The 35 values of eccentricity, each with an assumed standard deviation of 0.00004, were fitted with equation (11), in integrated form, using THROE. The zonal-harmonic perturbations are allowed for within THROE on this occasion; the lunisolar perturbations were pre-calculated, using PROD[8], but were found to be at most 0.7 times the standard deviation and generally far less, so they were ignored. The density scale height, H, was taken as 60km, appropriate to a height of 460km (3H/2 above perigee), and Λ was again taken as 1.1rev/day.

The same pairs of values of (γ,q) were used as with the i fitting, namely (1,0), (1,1) and (1,-1). The first run revealed that there was a clear oscillation in the US Navy values of e. So an attempt was made to reduce the oscillation by altering the value of J_3 used in removing the odd harmonic pertubations. By running THROE with $(\gamma,q) = (0,1)$ terms added, the best value of J_3 was found to be -3.066×10^{-6}, instead of the standard value, -2.531×10^{-6}. The use of this value of J_3 greatly improved the fit, reducing ε from 3.61 to 1.78. A further run was computed, omitting the same five values as on the final THROE run for i, for the same reasons. This improved the fit, giving $\varepsilon = 1.335$. For all the fittings of e, the values of $(\bar{C},\bar{S})_{29}^{0,2}$ were expected to be indeterminate because they are coefficients of small terms (see equation (11)), and only included so that the analysis of e conforms with that of i.

The 30 values of eccentricity, cleared of zonal-harmonic perturbations, are plotted in Fig.3 and the theoretical curve given by the final THROE run for e is drawn as a broken line. The values of the C and S coefficients obtained from the final THROE run are:

$$\left.\begin{aligned} 10^6\bar{S}_{29}^{0,2} &= 680 \pm 1080 & \qquad 10^6\bar{C}_{29}^{0,2} &= 1860 \pm 1010 \\ 10^6\bar{C}_{29}^{1,1} &= -37 \pm 12 & \qquad 10^6\bar{S}_{29}^{1,1} &= -26 \pm 17 \\ 10^6\bar{C}_{29}^{-1,3} &= 7.6 \pm 6.6 & \qquad 10^6\bar{S}_{29}^{-1,3} &= -13.7 \pm 5.5 \; . \end{aligned}\right\} \qquad (13)$$

The last four values agree with the last four in equations (12) to within twice the sum of their standard deviations, but the first two values in equations (13) are obviously unreliable, having standard deviations 2000 times greater than the corresponding values in (12). An extra run was computed without the first two terms, i.e. with $(\gamma,q) = (1,1)$ and $(1,-1)$, and the values were similar to (13), namely:

$$\begin{aligned} 10^6\bar{C}_{29}^{1,1} &= -37 \pm 11 & \qquad 10^6\bar{S}_{29}^{1,1} &= -23 \pm 15 \\ 10^6\bar{C}_{29}^{-1,3} &= 9.2 \pm 5.7 & \qquad 10^6\bar{S}_{29}^{-1,3} &= -15.7 \pm 5.4 \; . \end{aligned}$$

This confirms that the $(\gamma,q) = (1,0)$ terms had little influence on the results.

3.4 Inclination and eccentricity fitted simultaneously

The values of i and e were fitted simultaneously using the computer program SIMRES[4]. With this program there is a choice of weighting and two alternatives were tried, first with i and e having the same weight, and second with e degraded by a factor equal to the ratio of the final values of ε on the THROE fittings, namely 1.176 (= 1.335/1.135). The second fitting gave lower standard deviations for 5 of the 6 constants, and is more logical since the pre-assigned accuracies in i and e were arbitrary: the second fitting was therefore preferred. The fitting is shown in Figs.2 and 3 by unbroken lines and the values of the coefficients given by SIMRES are:

$$\left.\begin{array}{llll} 10^6\bar{S}_{29}^{0,2} = 1.1 \pm 0.5 & & 10^6\bar{C}_{29}^{0,2} = -1.0 \pm 0.5 & \\ 10^6\bar{C}_{29}^{1,1} = -6.6 \pm 2.7 & & 10^6\bar{S}_{29}^{1,1} = -7.5 \pm 4.6 & \\ 10^6\bar{C}_{29}^{-1,3} = 5.5 \pm 1.7 & & 10^6\bar{S}_{29}^{-1,3} = -0.2 \pm 1.4 & . \end{array}\right\} \qquad (14)$$

All these six values agree with those obtained from analysis of i alone, as given in equations (12), to within about 1/3 of a standard deviation, so the i-fitting is dominant. The variation of e given by SIMRES, as shown by the unbroken line in Fig.3, differs considerably from the THROE fitting, but is quite acceptable. The SIMRES fitting of i is of course similar to the THROE fitting, as Fig.2 shows.

For Ariel 1 the numerical versions of equations (7), (8) and (9) are:

$$\bar{C}_{29}^{0,2} = \bar{C}_{30,29} - 5.0\bar{C}_{32,29} + 11.5\bar{C}_{34,29} - 14.0\bar{C}_{36,29} + 6.8\bar{C}_{38,29} + 3.7\bar{C}_{40,29} - 5.4\bar{C}_{42,29} - 0.8\bar{C}_{44,29} + 3.4\bar{C}_{46,29} + \ldots\ldots \qquad (15)$$

$$\bar{C}_{29}^{1,1} = \bar{C}_{29,29} - 11\bar{C}_{31,29} + 44\bar{C}_{33,29} - 92\bar{C}_{35,29} + 107\bar{C}_{37,29} - 47\bar{C}_{39,29} - 36\bar{C}_{41,29} + 47\bar{C}_{43,29} + 9\bar{C}_{45,29} + \ldots\ldots \qquad (16)$$

$$\bar{C}_{29}^{-1,3} = \bar{C}_{29,29} - 9\bar{C}_{31,29} + 26\bar{C}_{33,29} - 39\bar{C}_{35,29} + 23\bar{C}_{37,29} + 10\bar{C}_{39,29} - 19\bar{C}_{41,29} - 2\bar{C}_{43,29} + 13\bar{C}_{45,29} + \ldots\ldots \qquad (17)$$

and similarly for S , on replacing C by S throughout.

Equations (15) to (17) show that the largest contributions to the lumped coefficients are likely to come from individual coefficients of degree 32-43. The expected order of magnitude of the lumped coefficients may be roughly estimated on the assumption that the individual coefficients of degree ℓ have numerical values of order $10^{-5}/\ell^2$, so that those with $32 < \ell < 43$ are of order 10^{-8}. The numerical coefficients in equation (16) are on average about eight times greater than those in equation (15), while the numerical coefficients in equation (17) are about three times greater than those in (15), so the magnitudes of the three successive lumped coefficients would be expected to be in the ratio 1:8:3. The actual ratios from equations (14) are $\sqrt{(1.1^2 + 1.0^2)}$: $\sqrt{(6.6^2 + 7.5^2)}$: $\sqrt{(5.5^2 + 0.2^2)}$ or 1:7:4 approximately; so the agreement is excellent.

From equations (15) to (17) the expected orders of magnitude of the numerical values of the three lumped coefficients are 0.2×10^{-6}, 1.5×10^{-6} and 0.5×10^{-6}. The actual values obtained in equations (14) are within a factor of 5 of the values indicated by this rough order-of-magnitude estimate, the actual values being generally the greater.

4 CONCLUSIONS

The perturbation noticed in the orbital inclination of Ariel 1 near 29:2 resonance has been analysed, together with the perturbation in eccentricity, to obtain, for the first time, values of lumped 29th-order harmonic coefficients in the geopotential. The values are given in equations (14), the symbols being defined in equations (1) to (9). The numerical values derived are not particularly accurate, but are as good as can be expected in view of the limited accuracy of the orbital data.

The success of this analysis shows that accurate values of 29th-order harmonics should be obtained in future, if satellites which pass through 29:2 resonance are intensively observed so that accurate orbits can be determined.

Acknowledgments

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REFERENCES

No.	Author	Title, etc.
1	D.G. King-Hele H. Hiller J.A. Pilkington	Table of Earth satellites, Volume 1: 1957 to 1968. RAE Technical Report 70020 (1970)
2	D.G. King-Hele D.M.C. Walker	Upper-atmosphere zonal winds: variation with height and local time. RAE Technical Report 76055 (1976)
3	W.M. Kaula	*Theory of satellite geodesy.* Blaisdell; Waltham, Mass. (1966)
4	R.H. Gooding	Studies of resonance in the orbits of Ariel satellites. RAE Technical Report (to be issued)
5	R.R. Allan	Resonance effect on inclination for close satellites. RAE Technical Report 71245 (1971) (Also *Planet. Space Sci.*, 21, 205-225 (1973))
6	J. Klokočnik	Changes of the inclination of a close Earth satellite due to orbital resonances. Astronomical Institute of Czechoslovak Academy of Sciences (to be published)
7	R.H. Gooding	Lumped geopotential coefficients $\bar{C}_{15,15}$ and $\bar{S}_{15,15}$ obtained from resonance variation in the orbit of Ariel 3. RAE Technical Report 71068 (1971)
8	G.E. Cook	PROD, a computer program for predicting the development of drag-free satellite orbits. Part 1: theory. RAE Technical Report 71007 (1971) *Celestial Mechanics*, 7, 301-314 (1973)

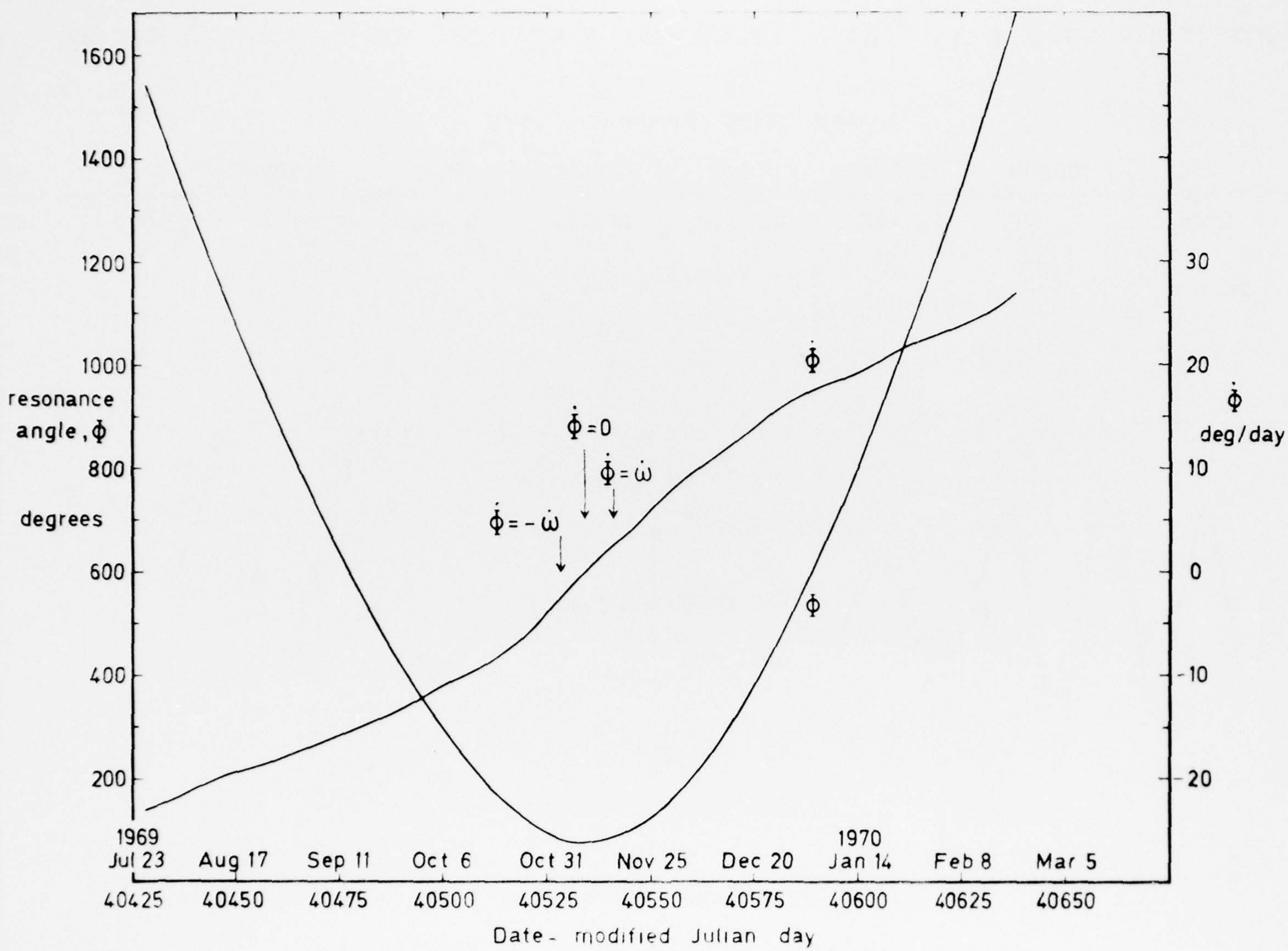

Fig.1 Variation of Φ and $\dot{\Phi}$

Fig.1

Fig. 2

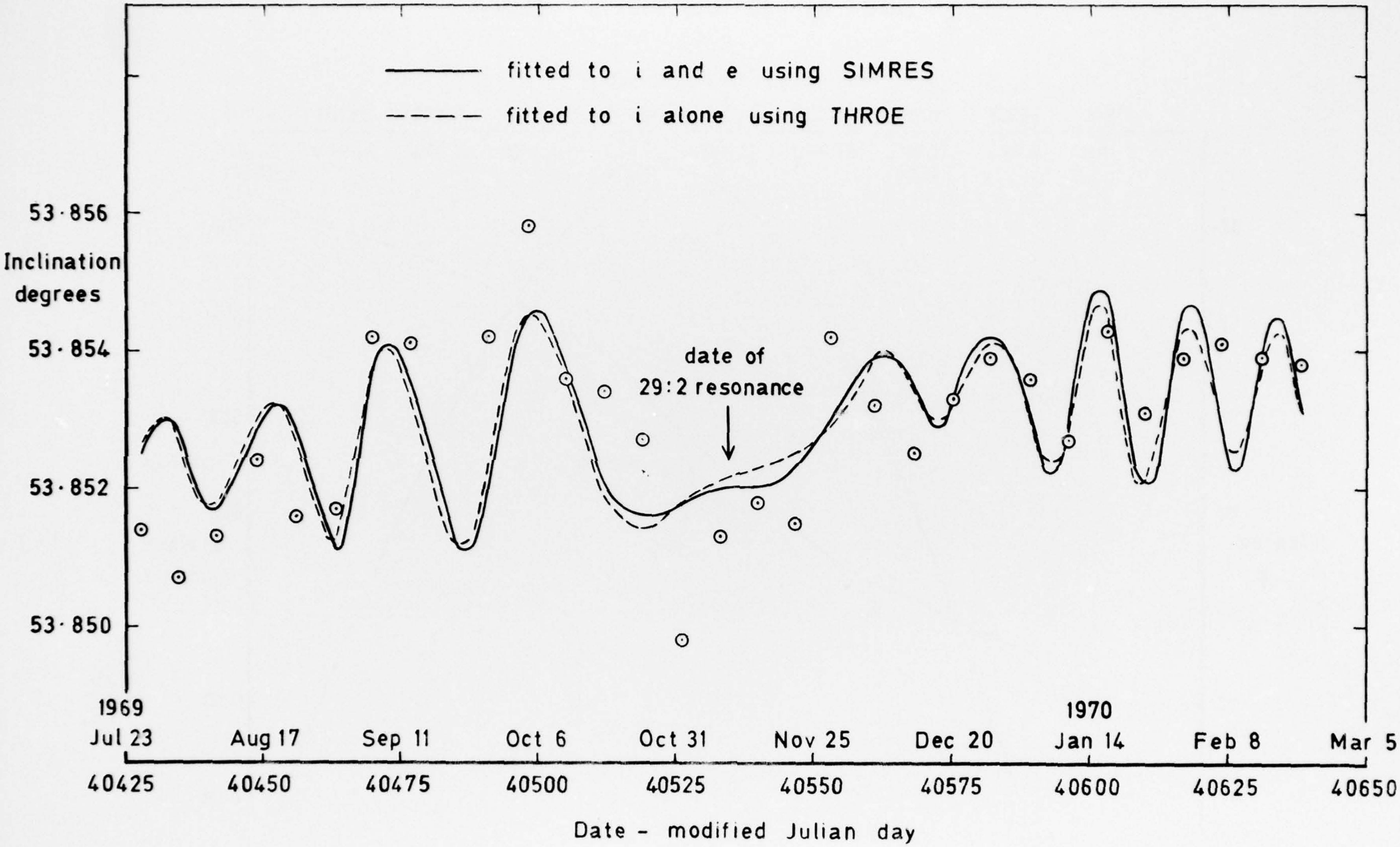

Fig. 2 Values of inclination near 29:2 resonance, with fitted theoretical curves

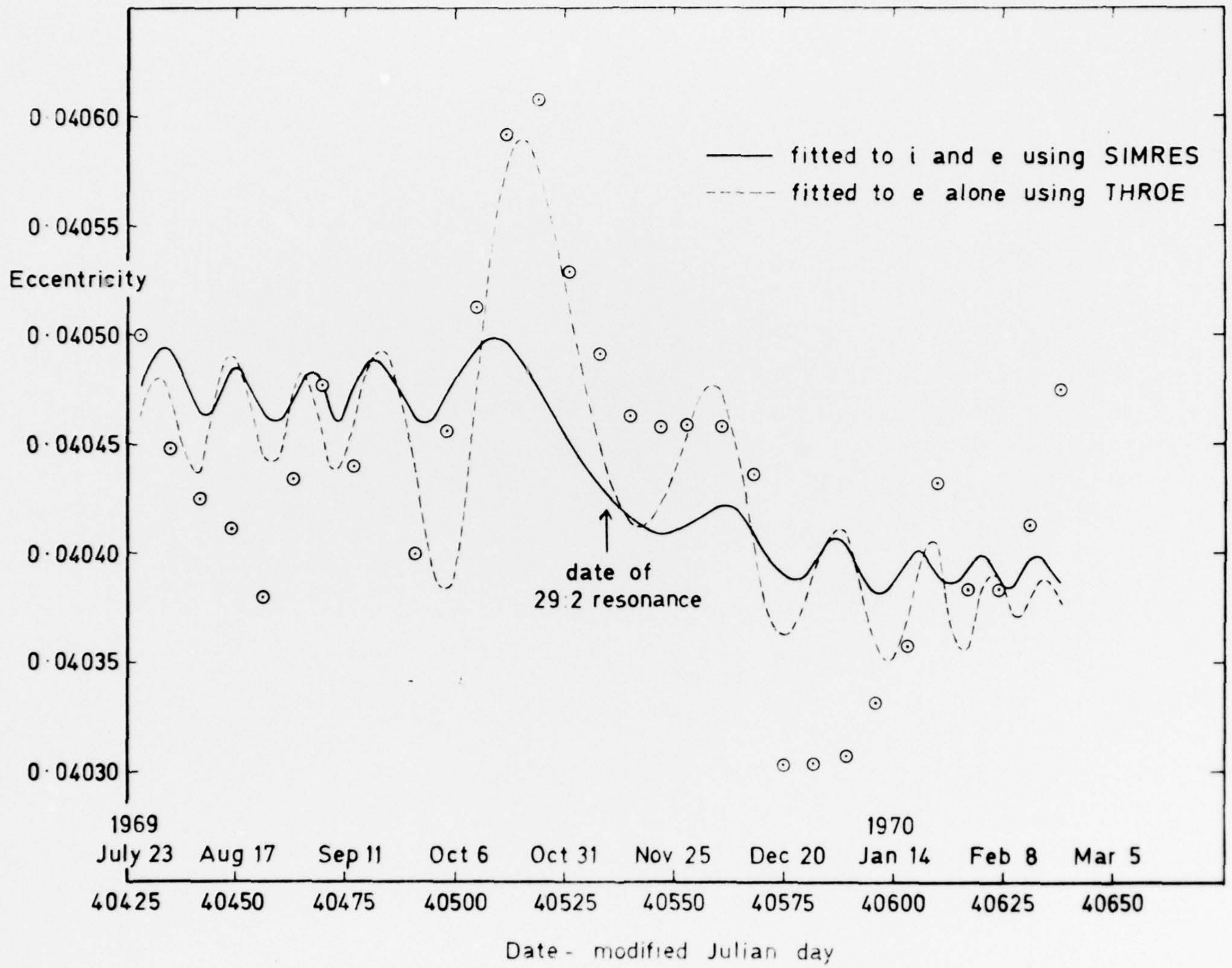

Fig.3 Values of eccentricity near 29:2 resonance, with fitted theoretical curves

Fig.3

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17. Abstract

In analysing the orbit of Ariel 1 to determine upper-atmosphere winds, it was observed that the orbital inclination underwent a noticeable perturbation in November 1969 at the 29:2 resonance with the Earth's gravitational field, when the satellite track over the Earth repeats every 2 days after 29 revolutions. The variations in the inclination and eccentricity of the orbit between July 1969 and February 1970 have now been analysed, using 35 US Navy orbits, and fitted with theoretical curves to obtain lumped values of 29th-order harmonic coefficients in the geopotential.

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